Grade 6 Primes and Composites

6.N.3	
Demonstrate an understanding of factors and multiples by • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems involving multiples.	1. Identify multiples for a number and explain the strategy used to identify them. [NOT DEVELOPED] 2. Determine all the whole-number factors of a number using arrays. [NOT DEVELOPED] 3. Identify the factors for a number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers or factor trees). [NOT DEVELOPED] 4. Provide an example of a prime number and explain why it is a prime number. [DEVELOPED] 5. Provide an example of a composite number and explain why it is a composite number. [DEVELOPED] 6. Sort a set of numbers as prime and composite. [DEVELOPED] 7. Solve a problem involving factors or multiples. [NOT DEVELOPED] 8. Explain why 0 and 1 are neither prime nor composite. [DEVELOPED]

Clarification of the outcome:

- ✦ The outcome concerns primes and not primes (specifically composites). There is more than one way to define a prime number. One way is by geometric thinking. If a number has more than two possible rectangles for it, that number is not prime. If a number has EXACTLY TWO possible rectangles for it, that number is prime. The numbers '0' and '1' are special (not prime or composite). Why? The DEVELOP lesson reveals why.
- ✦ The approach to defining a prime used here involves the number of ways to multiply. It does not involve division directly.
- ✦ The outcome is unpacked. Primes and composites are developed before factors and multiples because deep understanding of factors and multiples requires an understanding of primes and composites. In the DEVELOP lesson here, factors and multiples occur in a natural way but they are not identified or developed formally.

Required close-to-at-hand prior knowledge:

- ✤ Automaticity of multiplication facts.
- ✤ Understanding the array model for multiplication.
- ✤ Understanding models for multiplication (bag, rectangle/array).

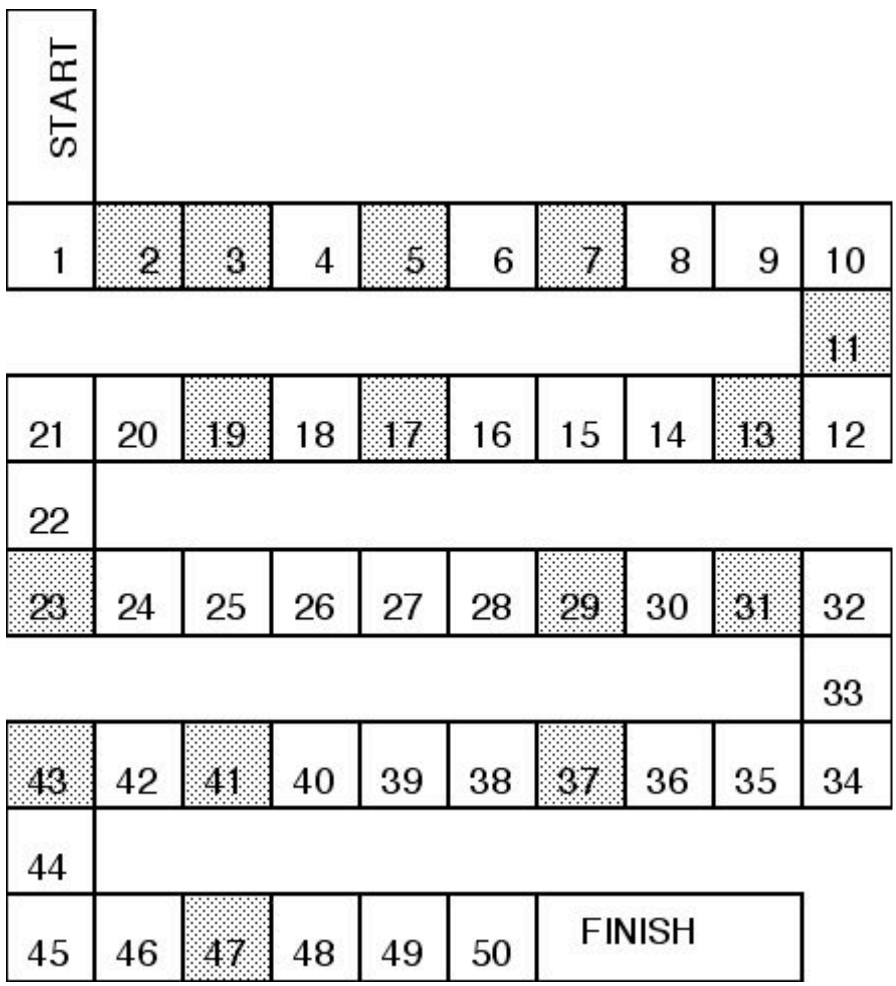
SET SCENE stage

The problem task to present to students:

Have students play the game, ***Go for 50***, for 10 to 15 minutes.

Game materials:

- A pair of normal six-sided dice (one die per team)
- An ordinary calculator
- A playing piece for each team
- The game board (see diagram)



Game rules:

- Two teams play at each game board. Each team consists of two players. [Will need to be adjusted if the number of students is not a multiple of 4.]
- Teams take turns playing the game. Each team has its own die (number cube).
- Each team begins the game at START.
- To determine the starting team, each team throws its die. The team throwing the higher number goes first.
- A team throws its die and moves the number of squares indicated on the die (e. g. if a 4 lands face up, the team's playing piece moves 4 squares forward). The team now has to figure out as many ways as they can think of for multiplying two whole numbers whose product is the number of the square they just landed on They get a point for each correct way. For example, if the team landed on square # 24, they might figure out these ways: 4 x 6 = 24, 6 x 4 = 24, 3 x 8 = 24, 8 x 3 = 24, 1 x 24 = 24, and 24 x 1 = 24. They figured out a total of 6 ways to multiply two whole numbers whose product of 24. The team would get 6 points. This point count is added to their existing point count. Then it is the other team's turn to play.
- The purpose of the calculator is to check any multiplication that might be challenged by the opposing team. The calculator cannot be used to figure out ways to get a product.
- The game is over when one of the teams goes past 50. A team does not have to throw the exact number to do that. It just needs to throw a number that moves its playing piece past 50. The winning team is the one that reaches 50 (or goes past) first.

Comments:

The game is designed to lead students to think informally about which numbers offer more points and which numbers offer fewer points. The shaded squares for all the prime numbers between 1 and 50 provide a visual cue for students about which numbers offer the least points.

DEVELOP stage

Activity 1: Revisits SET SCENE

- Discuss the ***Go for 50*** game. Ask students about numbers that offered fewer points and numbers that offered more points. Discuss feature of these numbers (e. g. more/less ways to multiply).
- Ask students what kinds of special numbers they know about. Expect at least; “odd and even numbers”. Suggest that there is another kind of special number, one that can be useful for doing mental arithmetic and for working with fractions. You might also want to mention that this special number has some interesting and unexpected uses in the real world.

[For example, prime numbers are at the heart of one way of encoding data so that it is almost impossible to decode it (secret message stuff). The technique relies on the enormous difficulty that there is in finding the prime factors of a very huge number (e. g. a 200 digit number). The technique was invented in a Winnipeg pub about 20 years ago by some engineers who were having a few beers.]

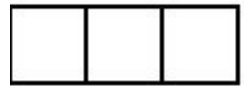
Activity 2: Addresses achievement indicators 4 and 5 informally.

- Select one small prime (a shaded number) and one small non-prime (a non-shaded number) found on the game board (e.g. prime: 5, non-prime: 12). Organize students into groups. Hand out sufficient unifix cubes to each group. Ask each group to make all possible equal piles for the selected prime (do not refer to it as a prime yet). For example, for the number '5', students would make one pile of 5 and five piles of 1.
- Ask each group to make all possible equal piles for the selected non-prime (do not refer to it as a non-prime yet). For example, for the number '12' students would make two piles of 6, six piles of 2, three piles of 4, four piles of 3, one pile of 12, and twelve piles of 1.
- Discuss how the two numbers are different from each other in relation to the number of ways of making piles (e.g. for 5, can only make two types of piles; for 12, can make six types of piles).
- Ask students how many ways there are to get the selected numbers (e.g. 5 and 12) as a product. Students should realize that there are only two ways to get each product (for example: 5 is 1 x 5 and 5 x 1). Discuss how this connects to making piles.
- Repeat for a different small prime and non-prime found on the game board.

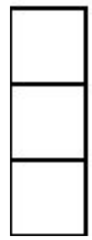
Activity 3: Addresses achievement indicators 4, 5 and, 6.

- ✦ Hand out grid paper. Split the class into 6 groups. Assign different numbers of the game board to each group. For example, group #1 works with the numbers from 2 to 8 (ignore '1' for the moment), group #2 works with the numbers from 9 to 16, etc. Ask each group to use the grid paper to build each of their numbers by making all possible rectangles whose dimensions are whole numbers and whose area (the number of squares in the rectangle) equals the number they are working with.

A 1 × 3 rectangle whose area = 3 squares.



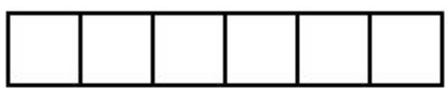
A 3 × 1 rectangle whose area = 3 squares.



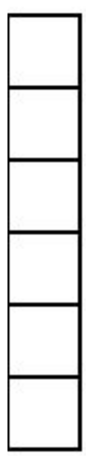
[The rectangles for 3 and 6 are shown here.]

- ✦ Put the data from the groups on the blackboard. Circle the numbers that were shaded on the game board (the primes). Ask students if they notice anything about the circled numbers. They should be able to tell you that each of the circled numbers are built by exactly two rectangles. All other numbers are built by more than 2 rectangles. Tell students the names: the circled numbers are primes and the rest are non-primes.

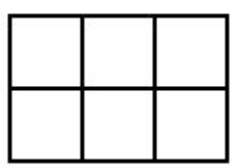
A 1 × 6 rectangle whose area = 6 squares.



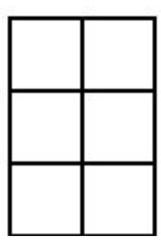
A 6 × 1 rectangle whose area = 6 squares.



A 2 × 3 rectangle whose area = 6 squares



A 3 × 2 rectangle whose area = 6 squares.



Activity 4: Addresses achievement indicators 4, 5 and, 6.

- ✦ Ask students to make a Venn diagram showing the primes and the non-primes on the game board. [Only one circle is needed. There is only one attribute involved – primeness. The Venn diagram should have all the primes inside the circle and the non-primes outside of it OR all of the primes outside the circle and the non-primes inside.]
- ✦ Ask students to write a definition of a prime number. Discuss their definitions.
- ✦ Ask students to explain why the number ‘2’ is the only even prime.
- ✦ Ask students to explain why not all odd numbers are prime.

Activity 5: Addresses achievement indicators 4, 5 and, 6.

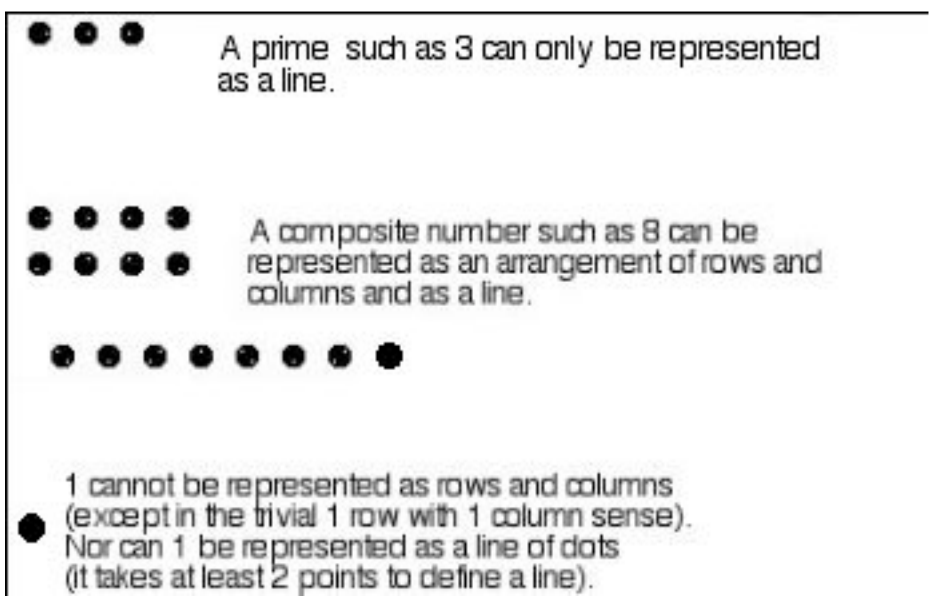
- ✦ Refresh the concept of prime by asking students to list a few prime numbers and to explain why they are prime.
- ✦ Ask students to invent a name for the non-primes on the game board (4, 6, 8, 9, 10, etc.). Discuss their inventions. Tell them the word that adults use for the non-primes is ‘composites’. Discuss why that word is reasonable (relate to’ compose’ - made up of). Ask students to explain the distinction between primes and composites.

Activity 6: Addresses achievement indicators 4, 5, 6, and 8.

- Ask students what number on the game board has not yet been named as either prime or composite. They should tell you the number ‘1’ has not yet been named.
- Ask them how many rectangles are possible to make for a prime and for a composite. [Expect two and more than two.]
- Ask them to make all possible rectangles for the number ‘1’. [Only one can be made.]
- Ask students to explain why the number ‘1’ is not prime or composite. They should tell you that you can make exactly two rectangles for any prime number, more than 2 rectangles for any composite number, and only one rectangle for the number ‘1’. So ‘1’ is not prime or composite.
- Tell students that 1 is a special number. It is sort of like the Adam and Eve of all numbers.

Note:

Another way to help students understand that 1 is special is to have them use dots to represent primes and composites. They should realize that primes can only be represented in one way - by a line (technically could be seen as a row having 'n' columns but the simplest way to look at the arrangement is as a line). They should realize that composites can be represented in two ways - a line and rows and columns. They should realize that 1 cannot be represented by a line or by a row and column. [Refer to the diagram.]



Activity 7: Addresses achievement indicators 4, 5, 6, and 8.

- Organize students into groups. Pose the question: Is '0' prime, composite or neither? Ask each group to answer the question and defend their decision with an explanation. [Some groups might decide that '0' is composite because it is not prime or because it has many multipliers (e.g. 0 is 2 x 0 or 3 x 0, etc.)] Lead students to realize that '0' is neither prime or composite by definition. Composites are defined as any positive whole number that has more than two numbers that multiply to give the number. The key word is positive: '0' is neither negative or positive.

Activity 8: Addresses achievement indicators 4, 5, 6, and 8.

- Provide students with a 1 to 50 number grid. Discuss how the gird is like a sieve. When you scratch out non-primes you are left with the primes.
- Ask them to scratch out the number ‘1’ and to explain why that is okay to do. Ask them to go to the first prime, 2, and scratch out all the numbers from 2 on, skip counting by 2. [2. 4. 6. 8. . . . would be scratched out.] Ask them to explain why that removes composites. Ask them to go to the next number that is not scratched out (the 3). Ask them to start at 3, and scratch out all the numbers from 3 on, skip counting by 3. Ask students to go to the next number that is not scratched out (the 5). Ask them to start at 5, and scratch out all the numbers from 5 on, skip counting by 5. Ask them to continue in this way until they have scratched out all composites (and ‘1’), leaving behind the primes.

Here is the grid after the skip counting by 5 is complete. Notice that some numbers have more than one scratch out mark on them (because they are multiples of 2 and of 3 or multiples of 2 and 5, or etc.).

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	25	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	35	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49	~~50~~

- Ask students to compare the numbers not scratched out to the shaded numbers on the game board. They should notice the two sets of numbers are the same.

Activity 9: Addresses achievement indicators 4, 5, 6, and revisits SET SCENE.

- Ask students to make a game board for a new game ***Go for 100***. The board should have all the prime numbers shaded in.
- After the board has been completed, have students play ***Go for 100*** for 10 to 15 minutes.

Activity 10: Assessment of teaching.

- Provide students with the following list of numbers: 0, 1, 2, 9, 13, 20, 21, 29, 38, 41.
- Ask them to circle all the primes.
- Ask them to select one of the circled primes and explain why it is a prime.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partial well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheet.

Question 1.

There are two whole numbers that are neither prime or composite. What are they. Explain why they are neither prime or composite.

Question 2.

List all the primes less than 30.

Question 3.

Write three consecutive composite numbers.

Question 4.

I am a prime less than 20. If you add 20 to me, the result is prime. What number am I?

Question 5.

What is the sum of the first 5 prime numbers?

Question 6.

What is the sum of the first 5 composite numbers?

MAINTAIN stage

Mini-task example

Every so often:

- Provide students with a couple of numbers (between 100 and 200). Ask them to decide whether the numbers are prime of composite.

Rich-task example #1

If repetition of a prime number is allowed, then every even number greater than 2 can be expressed as the sum of exactly two prime numbers. Here are some examples: 12 = 7 + 5, 4 = 2 + 2, 6 = 3 + 3, 8 = 5 + 3, 50 = 31 + 19. Ask students to express all even numbers greater than 2 and less than 80 as the sum of two primes.

Rich-task example #2

There is a subset of primes called twin primes. Some examples are: 3 and 5, 5 and 7, 11 and 13, 17 and 19, etc. Twin primes are always consecutive odd numbers that also happen to be prime. Ask students to investigate whether there is a pattern involved in forming twin primes. They should notice this pattern: 3 = 2 x 2 - 1 and 5 = 2 x 2 + 1; 5 = 2 x 3 - 1 and 7 = 2 x 3 + 1; 11 = 3 x 4 - 1 and 13 = 3 x 4 + 1; etc.

Comments

These are rich-tasks because they involve complex problem solving and integrate other mathematical concepts.